ON THE MOTION OF RIGID-PLASTIC BODIES IN A RESISTING MEDIUM

(O DVIZHENII ZHESTKO-PLASTICHESKIKH TEL V SOPROTIVLIAIUSHCHEISIA SREDE)

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The effects of a resisting medium on the motion of rigid-plastic bodies can be easily accounted for in the case when the resistance of a medium depends on the velocities of the points of rigid-plastic structures. As an example there is considered the case of motion of an infinite beam subjected to an impact loading applied with constant velocity. The resistance of the medium is assumed to be a power function of the velocities.

The results of this investigation may be extended to the case of rigid-plastic plates and shells moving in a resisting medium.

A rigorous formulation of the problem of motion of a rigid-plastic body moving in a resisting medium, requiring considerations of the motion of both the body and the medium, is very complicated. Substantial simplification is obtained for the case when the law of the resistance of the medium is given a priori. Such an approach was used in the past to solve a number of problems. Thus, in the case of beams and plates resting on an elastic foundation, it is assumed that the resisting force of the foundation is proportional to the displacements of the beam, (cf. [1]; an analogous assumption was also made for rigid-plastic beams [2]). In studies of the action of impulsive waves on ship structures it is sometimes assumed that the resistance of the medium is proportional to the first power of the velocities of the bending deformations. Such an assumption was made for cylindrical shells [3], and for an initial motion of plates [4]. An assumption regarding the resistance of the medium being proportional to the square of the velocities of the displacements was made in the investigations of the effects of transverse impact loading on rigid-plastic strings and membranes [5,6].

In the solution of the dynamic problem for rigid-plastic structures it is necessary to assume the velocity fields of the displacements, which must be compatible with the flow law and the plasticity law of the plastic potential. If the plastic hinges or the flow lines are stationary, then the problem of the determination of the velocity field is equivalent to the problem of the determination of the displacement field. Moreover, in this case the form of the solution will be independent from the characterization of the resistance of the medium either by the velocities or by the displacements. If, however, the plastic hinges or the flow lines are not stationary, then simple solutions are obtained assuming that the resistance of the medium depends on the velocities only.

As an example, let us consider the problem of an infinite beam subjected to a load applied with constant velocity. The solution of this problem without the resisting forces is known [7, 8]. Let the axis of the beam coincide with the x-axis of the coordinate system. Assume also that the resisting forces r depend on the velocity y as follows

$$r = \alpha \dot{y}^n \tag{1}$$

where α is a constant multiplier. Note that for $n = 1$, i.e. in the case of the waves acting on the ship structure, it is assumed that $\alpha = \rho c$, where ρ is the density and c is the speed of sound [3, 4]. We assume, moreover, that the velocity field is given in the form, [8]

$$\dot{y} = \begin{cases} v_0 [1 - x / \xi(t) & (0 \leqslant x \leqslant \xi(t)) \\ 0 & (x \geqslant \xi(t)) \end{cases} \tag{2}$$

Here $\xi(t)$ is the coordinate of a stationary plastic hinge. It is assumed that at the point of impact a stationary plastic hinge is formed and two nonstationary plastic hinges are moving in opposite directions from the stationary hinge.

The equation of motion of a beam is

$$M'' = r + m\ddot{y} \tag{3}$$

Using (1) and (2) and integrating (3), taking into account that $M'(0, t) = P/2$, where P is the external force acting on the beam, we have

$$M' = -\frac{P}{2} + \alpha v_0{}^n x \left[1 - \frac{n}{2!}\left(\frac{x}{\xi}\right) + \frac{n(n-1)}{3!}\left(\frac{x}{\xi}\right)^2 - \dots\right] + \frac{m v_0 \dot{\xi} x^2}{2\xi^2} \tag{4}$$

Integrating this equality and noticing that $M(0, t) = M_0$ we have

$$M = M_0 - \frac{P}{2}x + \alpha v_0{}^n x^2 \left[\frac{1}{2} - \frac{n}{3!}\left(\frac{x}{\xi}\right) + \frac{n(n-1)}{4!}\left(\frac{x}{\xi}\right)^2 - \dots\right] + \frac{m v_0 \dot{\xi} x^3}{6\xi^2} \tag{5}$$

The condition $M'(\xi, t) = 0$ results in

$$-\frac{P}{2}+\alpha v_0^n\xi\left[1-\frac{n}{2!}+\frac{n(n-1)}{3!}-\dots\right]+\frac{mv_0\dot{\xi}}{2}=0 \tag{6}$$

and from the condition $M(\xi, t) = -M_0$ we have

$$2M_0-\frac{P}{2}\xi+\alpha v_0^n\xi^2\left[\frac{1}{2!}-\frac{n}{3!}+\frac{n(n-1)}{4!}-\dots\right]+\frac{mv_0\dot{\xi}\xi}{6}=0 \tag{7}$$

From (6) and (7) we obtain differential equation for ξ

$$\xi\dot{\xi}+\frac{3\alpha v_0^{n-1}\sigma(n)}{n}\xi^2=\frac{6M_0}{mv_0}\qquad\left(\sigma(n)=\frac{1}{2!}-\frac{2n}{3!}+\frac{3n(n-1)}{4!}-\dots\right) \tag{8}$$

The solution of this equation has the form

$$\xi=\left\{\frac{2M_0}{\alpha v_0^n\sigma(n)}\left(1-\exp\left[-\frac{6\alpha v_0^{n-1}\sigma(n)}{m}t\right]\right\}^{1/2} \tag{9}$$

The solution for the case without the resisting force can be obtained from (9) by letting $\alpha \to 0$. We thus find

$$\xi=\left(\frac{12M_0t}{v_0m}\right)^{1/2} \tag{10}$$

This formula coincides with the solution obtained by Conroy [7] and Hopkins [8].

Comparison of (9) and (10) shows the difference in the behavior of a beam caused by the presence of the resisting forces.

For $t \to \infty$ the whole free beam will be affected by the motion, while in the presence of resisting forces the perturbation will affect only the following segment of the beam

$$\xi=\frac{2M_0}{\alpha v_0^n\sigma(n)}$$

From (9) we have

$$\xi=\left(\frac{12M_0}{\alpha v_0}\left[1-\exp\left(-\frac{\alpha t}{m}\right)\right]\right)^{1/2}\quad\text{for } n=1 \tag{11}$$

$$\xi=\left\{\frac{24}{7\alpha v_0^2}\left[1-\exp\left(-\frac{7}{2}\frac{\alpha v_0t}{m}\right)\right\}^{1/2}\quad\text{for } n=2 \tag{12}$$

Using (6) or (7) in connection with (9), we can see that, similarly as in the case without the resisting force, at the initial instant the force P has a singularity of the form $t^{-1/2}$. Moreover, one must be assured that for $0\leqslant x\leqslant\xi$ the condition $|M|\leqslant M_0$ is satisfied. This fact is easily verified, at least for $n = 1$ and $n = 2$, through an analysis of the Formulas (4) and (6).

The solution (9) was obtained assuming that the force P is acting on the beam. It can be demonstrated that at the removal of the force *P* the whole beam must return to such a state of motion which would be independent of the existence of the resisting force. This was demonstrated in [8] assuming that there are no resisting forces.

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